

Exercise 2.1.3

In the next three exercises, interpret $\dot{x} = \sin x$ as a flow on the x -axis.

(Acceleration)

- a) Find the flow's acceleration \ddot{x} as a function of x .
 - b) Find the points where the flow has maximum positive acceleration.
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Solution

Differentiate both sides with respect to t .

$$\begin{aligned}\frac{d}{dt}(\dot{x}) &= \frac{d}{dt}(\sin x) \\ \ddot{x} &= (\cos x) \cdot \frac{d}{dt}(x) \\ &= (\cos x) \cdot \dot{x} \\ &= (\cos x)(\sin x) \\ &= \frac{1}{2} \sin 2x\end{aligned}$$

The greatest acceleration to the right occurs where \ddot{x} is maximum (and positive), that is, where $\sin 2x = 1$:

$$\begin{aligned}2x &= \frac{\pi}{2} + 2n\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ x &= \frac{\pi}{4} + n\pi.\end{aligned}$$